Lesson 1.1 Reteach *Rates*

A ratio that compares two quantities with different kinds of units is called a **rate**. When a rate is simplified so that it has a denominator of 1 unit, it is called a **unit rate**.

Example 1

DRIVING Alita drove her car 78 miles and used 3 gallons of gas. What is the car's gas mileage in miles per gallon?

Write the rate as a fraction. Then find an equivalent rate with a denominator of 1.

78 miles using 3 gallons $= \frac{78 \text{ mi}}{3 \text{ gal}}$	Write the rate as a fraction.
$= \frac{78 \text{ mi} \div 3}{3 \text{ gal} \div 3}$	Divide the numerator and the denominator by 3.
$= \frac{26 \text{ mi}}{1 \text{ gal}}$	Simplify.

The car's gas mileage, or unit rate, is 26 miles per gallon.

Example 2

SHOPPING Joe has two different sizes of boxes of cereal from which to choose. The 12-ounce box costs \$2.54, and the 18-ounce box costs \$3.50. Which box costs less per ounce?

Find the unit price, or the cost per ounce, of each box. Divide the price by the number of ounces.

12-ounce box	\$2.54 ÷ 12 ounces ≈ \$0.21 per ounce
18-ounce box	\$3.50 ÷ 18 ounces ≈ \$0.19 per ounce

The 18-ounce box costs less per ounce.

Lesson 1.2 Reteach *Complex Fractions and Unit Rates*

Fractions like $\frac{2}{\frac{3}{4}}$ are called complex fractions. **Complex fractions** are fractions with a numerator, denominator, or both that are also fractions.

Example 1

Simplify $\frac{2}{\frac{3}{4}}$.

A fraction can also be written as a division problem.

$\frac{2}{\frac{3}{4}} = 2 \div \frac{3}{4}$	Write the complex fraction as a division problem.
$= \frac{2}{1} \times \frac{4}{3}$	Multiply by the reciprocal of $\frac{3}{4}$ which is $\frac{4}{3}$.
$= \frac{8}{3}$ or $2\frac{2}{3}$	Simplify.

So, $\frac{2}{\frac{3}{4}}$ is equal to $2\frac{2}{3}$.

Lesson 1.3 Reteach — *Convert Unit Rates*

Unit ratios and their reciprocals can be used to convert rates. Sometimes you have to multiply more than once.

Example

The speed limit on the interstate is 65 miles per hour. How many feet per minute is the speed limit?

Because the unit of miles must divide out, use the unit ratio $\frac{5{,}280\text{ ft}}{1\text{ mi}}$ because the unit of miles is in the denominator. Use $\frac{1\text{ h}}{60\text{ min}}$ to convert from hours to minutes.

$$\frac{65\text{ mi}}{1\text{ h}} = \frac{65\text{ mi}}{1\text{ h}} \cdot \frac{5{,}280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ h}}{60\text{ min}}$$ Multiply by the appropriate ratios.

$$= \frac{65\text{ mi}}{1\text{ h}} \cdot \frac{5{,}280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ h}}{60\text{ min}}$$ Divide out common units.

$$= \frac{65 \cdot 5{,}280\text{ ft} \cdot 1}{1 \cdot 1 \cdot 60\text{ min}} = \frac{343{,}200\text{ ft}}{60\text{ min}} \text{ or } \frac{5{,}720\text{ ft}}{1\text{ min}}$$ Simplify.

The speed limit is 5,720 feet per minute.

Lesson 1.4 Reteach — *Proportional and Nonproportional Relationships*

Two related quantities are **proportional** if they have a constant ratio between them. If two related quantities do not have a constant ratio, then they are **nonproportional**.

Example 1

The cost of one CD at a record store is $12. Create a table to show the total cost for different numbers of CDs. Is the total cost proportional to the number of CDs purchased?

Number of CDs	1	2	3	4
Total Cost	$12	$24	$36	$48

$$\frac{\text{Total Cost}}{\text{Number of CDs}} = \frac{12}{1} = \frac{24}{2} = \frac{36}{3} = \frac{48}{4} = \$12 \text{ per CD}$$ Divide the total cost for each by the number of CDs to find a ratio. Compare the ratios.

Since the ratios are the same, the total cost is proportional to the number of CDs purchased.

Example 2

The cost to rent a lane at a bowling alley is $9 per hour plus $4 for shoe rental. Create a table to show the total cost for each hour a bowling lane is rented if one person rents shoes. Is the total cost proportional to the number of hours rented?

Number of Hours	1	2	3	4
Total Cost	$13	$22	$31	$40

$$\frac{\text{Total Cost}}{\text{Number of Hourse}} \rightarrow \frac{13}{1} \text{ or } 13 \quad \frac{22}{2} \text{ or } 11 \quad \frac{31}{3} \text{ or } 10.34 \quad \frac{40}{4} \text{ or } 10$$ Divide each cost by the number of hours.

Since the ratios are not the same, the total cost is nonproportional to the number of hours rented with shoes.

Lesson 1.5 Reteach

Graph Proportional Relationships

A way to determine whether two quantities are proportional is to graph them on a coordinate plane. If the graph is a straight line through the origin, then the two quantities are proportional.

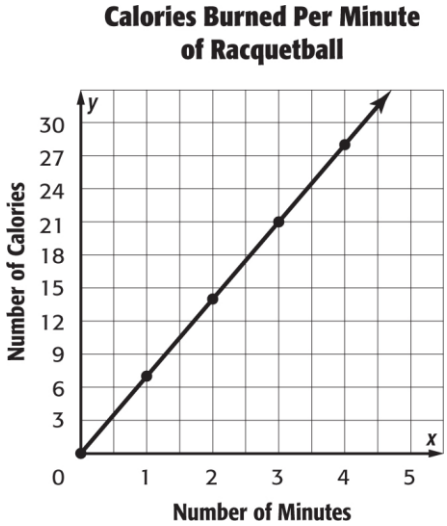
Example 1

A racquetball player burns 7 Calories a minute. Determine whether the number of Calories burned is proportional to the number of minutes played by graphing on the coordinate plane.

Step 1 Make a table to find the number of Calories burned for 0, 1, 2, 3, and 4 minutes of playing racquetball.

Time (min)	0	1	2	3	4
Calories Burned	0	7	14	21	28

Step 2 Graph the ordered pairs on the coordinate plane. Then connect the ordered pairs.



The line passes through the origin and is a straight line. So, the number of Calories burned is proportional to the number of minutes of racquetball played.

Lesson 1.6 Reteach

Solve Proportional Relationships

A **proportion** is an equation that states that two ratios are equivalent. To determine whether a pair of ratios forms a proportion, use cross products. You can also use cross products to solve proportions.

Example 1

Determine whether the pair of ratios $\frac{20}{24}$ and $\frac{12}{18}$ form a proportion.

Find the cross products.

$\frac{20}{24} \stackrel{?}{=} \frac{12}{18}$ → $24 \bullet 12 = 288$

→ $20 \bullet 18 = 360$

Since the cross products are not equal, the ratios do not form a proportion.

Example 2

Solve $\frac{12}{30} = \frac{k}{70}$.

$\frac{12}{30} = \frac{k}{70}$	Write the equation.
$12 \cdot 70 = 30 \cdot k$	Find the cross products.
$840 = 30k$	Multiply.
$\frac{840}{30} = \frac{30k}{30}$	Divide each side by 30.
$28 = k$	Simplify.

The solution is 28.

Lesson 1.7 Reteach *Constant Rate of Change*

A **rate of change** is a rate that describes how one quantity changes in relation to another.
A **constant rate of change** is the rate of change of a linear relationship.

Example 1

Find the constant rate of change for the table.

Students	Number of Textbooks
5	15
10	30
15	45
20	60

The change in the number of textbooks is 15. The change in the number of students is 5.

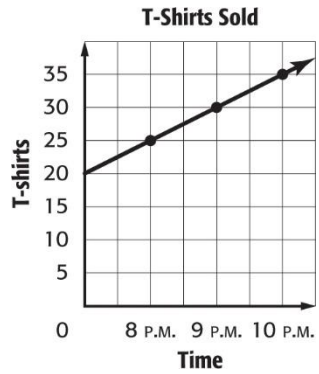
$$\frac{\text{change in number of textbooks}}{\text{change in number of students}} = \frac{15 \text{ textbooks}}{5 \text{ students}}$$ The number of textbooks increased by 15 for every 5 students.

$$= \frac{3 \text{ textbooks}}{1 \text{ students}}$$ Write as a unit rate.

So, the number of textbooks increases by 3 textbooks per student.

Example 2

The graph represents the number of T-shirts sold at a band concert. Use the graph to find the constant rate of change in number per hour.



To find the rate of change, pick any two points on the line, such as (8, 25) and (10, 35).

$$\frac{\text{change in number}}{\text{change in time}} = \frac{(35-25)}{(10-8)} = \frac{10}{2} \text{ or 5 T-shirts per hour}$$

Lesson 1.8 Reteach *Slope*

Slope is the rate of change between any two points on a line.

$$\text{slope} = \frac{\text{change in y}}{\text{change in x}} = \frac{\text{vertical change}}{\text{horizontal change}} \text{ or } \frac{\text{rise}}{\text{run}}$$

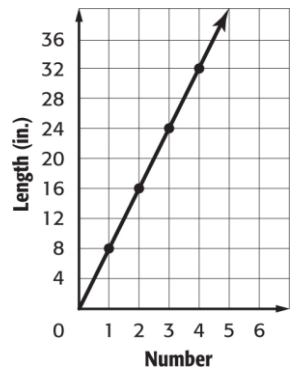
Example

The table shows the length of a patio as blocks are added.

Number of Patio Blocks	0	1	2	3	4
Length (in.)	0	8	16	24	32

Graph the data. Then find the slope of the line.

Explain what the slope represents.



$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$ Definition of slope

$= \frac{24 - 8}{3 - 1}$ Use $(1, 8)$ and $(3, 24)$.

$= \frac{16}{2}$ $\frac{\text{length}}{\text{number}}$

$= \frac{8}{1}$ Simplify.

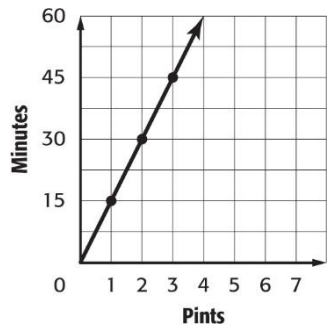
So, for every 8 inches, there is 1 patio block.

Lesson 1.9 Reteach *Direct Variation*

When two variable quantities have a constant ratio, their relationship is called a **direct variation**.
The constant ratio is called the **constant of proportionality**.

Example 1

The time it takes Lucia to pick pints of blackberries is shown in the graph. Determine the constant of proportionality.



Since the graph forms a line, the rate of change is constant.
Use the graph to find the constant of proportionality.

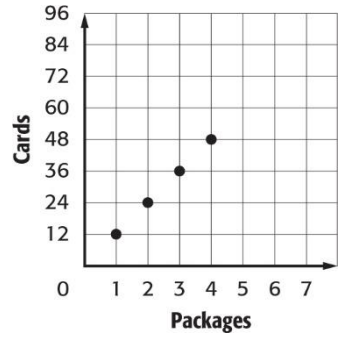
$\frac{\text{minutes}}{\text{number of pints}} = \frac{15}{1}$ $\frac{30}{2}$ or $\frac{15}{1}$ $\frac{45}{3}$ or $\frac{15}{1}$

It takes 15 minutes for Lucia to pick 1 pint of blackberries.

Example 2

There are 12 trading cards in a package. Make a table and graph to show the number of cards in 1, 2, 3, and 4 packages. Is there a constant rate? a direct variation?

Numbers of Packages	1	2	3	4
Number of Cards	12	24	36	48



Because there is a constant increase of 12 cards, there is a constant rate of change. The equation relating the variables is
$y = 12x$, where y is the number of cards and x is the number of packages. This is a direct variation. The constant of proportionality is 12.